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Gravitational radiation from corotating binary neutron stars of incompressible fluid in the first post-Newtonian approximation of general relativity

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We analytically study gravitational radiation from corotating binary neutron stars composed of incompressible, homogeneous fluid in circular orbits. The energy and the angular momentum loss rates are derived up to the first post-Newtonian (1PN) order beyond the quadrupole approximation including effects of the finite size of each star of binary. It is found that the leading term of finite size effects in the 1PN order is only $O(GM_*/c^2a_*)$ smaller than that in the Newtonian order, where GM_*/c^2a_* means the ratio of the gravitational radius to the mean radius of each star of binary, and the 1PN term acts to decrease the Newtonian finite size effect in gravitational radiation.

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I. INTRODUCTION

Around the beginning of the next century, the laser interferometers for detection of gravitational waves, such as LIGO [1], VIRGO [2], GEO600 [3] and TAMA300 [4] will be in operation. The most important targets for these detectors are coalescing binary neutron stars (BNS's) because in a period of their inspiraling phase they will emit gravitational waves of frequencies in the sensitive range of these detectors, i.e., from 10Hz to 1000Hz. We have to prepare accurate theoretical templates in order to extract informations of BNS's such as each mass, spin and so on using matched filtering technique [5].

In their early inspiraling phase, the hydrodynamical effect of each star of binary is less effective and the point particle approximation works well. In the point particle approximation, the energy loss rate is calculated up to 2.5 post-Newtonian (PN) order for arbitrary mass binaries [6]. The other approach for the inspiraling phase is the black hole perturbation method. In this formalism, the energy loss rate is calculated up to 4PN order in a Kerr black hole case [7], and 5.5PN order in a Schwarzschild black hole case [8].

In the late inspiraling phase, however, the hydrodynamical effects of each star of binary become important, and we have to study the evolution of the binary taking into account them [9] [10] [11] [12]. The importance of the study of the late inspiraling phase comes from the existence of the innermost stable circular orbit (ISCO). If we are able to know the location of the ISCO from the signal of gravitational waves, we can see a strong general relativistic phenomenon as well as we may get the information of the equation of state of neutron stars, i.e., the relation between the mass and the radius [13] [14]. To know the precise location of the ISCO, it is desirable to construct the theoretical template of gravitational waves in the late inspiraling phase as in the early one. Up to the Newtonian order, we have already understood gravitational radiation from binary systems of finite size stars [15] [16] [17], but apparently the Newtonian treatment is not appropriate because BNS's are general relativistic objects.

To investigate general relativistic effects on gravitational waves in the late inspiraling phase, in this paper, we analytically calculate the energy and angular momentum loss rates of the 1PN order including the finite size effects. In the calculation, we assume that BNS's are composed of the incompressible and homogeneous fluid and then obtain

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an ellipsoidal equilibrium configuration for each star. Also, we suppose the corotating circular orbit in which each star of binary uniformly rotates around the center of mass of the system. Although the assumption of corotation is not appropriate for the realistic one which will be almost irrotational [18] [17], the energy and angular momentum loss rates we calculate in this paper will be useful to see contribution of the finite size effects.

This paper is organized as follows. In Sec. II, we derive the mass quadrupole moments and their time derivatives which are necessary for calculation of gravitational waves by using Blanchet-Damour formalism [19]. In Sec. III, we give the rates of the energy and angular momentum loss gathering the time derivatives of the mass quadrupole moments which we derive in Sec. II, and their quantitative results are given in Sec. IV. Section V is devoted to summary and discussion.

Throughout this paper, we use the unit $G = 1$ and c denotes the speed of light. Latin indices $i,\ j,\ k, \cdots$ take 1 to 3, and δ_{ij} denotes the Kronecker's delta. We use I_{ij} and $\not{I}_{ij}$ as the quadrupole moment and its trace free part,

$$I_{ij} \equiv \int d^3x \rho x_i x_j = \frac{M}{5} a_i^2 \delta_{ij}, \tag{1.1}$$

$$\not{I}_{ij} = I_{ij} - \frac{1}{3}\delta_{ij} \sum_{k=1}^{3} I_{kk}, \tag{1.2}$$

where M and a_i denote the Newtonian mass and the principal axes of the star.

II. FORMULATION

In this paper, we calculate the energy and angular momentum loss rates of gravitational radiation from corotating binary stars. Each star of binary is assumed to be composed of the incompressible and homogeneous fluid because all the calculations can be done analytically. We pay particular attention to BNS's of equal masses in circular orbits whose coordinate separation is R. We assume that the center of mass of the binary system locates at the origin of the *inertial* coordinate system (X_1, X_2, X_3), and the center of mass of a star (star 1) locates at the origin of the *comoving* coordinate system (x_1, x_2, x_3). In the comoving coordinate system, the other star (star 2) locates at $(x_1, x_2, x_3) = (-R, 0, 0)$. The transformation from the coordinate system (X_1, X_2, X_3) to (x_1, x_2, x_3) is given by

$$\begin{aligned} X_1 &= \left(x_1 + \frac{R}{2}\right)\cos\Omega t - x_2 \sin\Omega t, \\ X_2 &= \left(x_1 + \frac{R}{2}\right)\sin\Omega t + x_2 \cos\Omega t, \\ X_3 &= x_3. \end{aligned} \tag{2.1}$$

According to Blanchet and Damour [19], in the 1PN order, the energy and angular momentum loss rates are written as

$$\frac{dE}{dt} = -\frac{1}{5c^5}\left[M_{ij}^{(3)} M_{ij}^{(3)} + \frac{1}{c^2}\left(\frac{5}{189} M_{ijk}^{(4)} M_{ijk}^{(4)} + \frac{16}{9} S_{ij}^{(3)} S_{ij}^{(3)}\right)\right], \tag{2.2}$$

$$\frac{dJ^3}{dt} = -\frac{2}{5c^5}\epsilon_{3jk}\left[M_{jl}^{(2)} M_{kl}^{(3)} + \frac{1}{c^2}\left(\frac{5}{126} M_{jlm}^{(3)} M_{klm}^{(4)} + \frac{16}{9} S_{jl}^{(2)} S_{kl}^{(3)}\right)\right], \tag{2.3}$$

where $M_{ij\cdots}$ and S_{ij} are Blanchet-Damour's mass multipole moments and the current quadrupole moment, and the superscript (n) where $n = 2,\, 3,\, 4$ denotes the time derivative d^n/dt^n. Since we assume that the binary rotates around the X_3 axis, there is only $i = 3$ component in the angular momentum loss rate. In the subsequent subsections, we derive $M_{ij\cdots}$ and S_{ij} which are necessary for calculation of gravitational waves.

A. Mass quadrupole moment

First, we calculate M_{ij} up to the 1PN order. M_{ij} is the even term of the inertial coordinates X_1, X_2, and X_3, and we consider the equal mass binary. Then, the contribution from star 1 is the same as that from star 2. M_{ij} is expressed as

$$
\begin{aligned}
M_{ij}(t) &= \int d^3X\sigma\hat{X}_{ij} + \frac{1}{14c^2}\frac{d^2}{dt^2}\biggl(\int d^3X\rho\hat{X}_{ij}\mathbf{X}^2\biggr) - \frac{20}{21c^2}\frac{d}{dt}\biggl(\int d^3X\rho v_k\hat{X}_{ijk}\biggr), \\
&= \int d^3X\rho\hat{X}_{ij} + \frac{1}{c^2}\biggl[\int d^3X\rho\biggl(2v^2+2U+\frac{3P}{\rho}\biggr)\hat{X}_{ij} + \frac{1}{14}\frac{d^2}{dt^2}\biggl(\int d^3X\rho\hat{X}_{ij}\mathbf{X}^2\biggr) \\
&\qquad -\frac{20}{21}\frac{d}{dt}\biggl(\int d^3X\rho v_k\hat{X}_{ijk}\biggr)\biggr],
\end{aligned}
\tag{2.4}
$$

where

$$
\sigma = \rho\biggl[1+\frac{1}{c^2}\biggl(2v^2+2U+\frac{3P}{\rho}\biggr)\biggr], \tag{2.5}
$$

$$
\hat{X}_{ij} = X_iX_j - \frac{1}{3}\delta_{ij}\mathbf{X}^2, \tag{2.6}
$$

$$
\hat{X}_{ijk} = X_iX_jX_k - \frac{1}{5}\mathbf{X}^2(\delta_{ij}X_k+\delta_{jk}X_i+\delta_{ki}X_j), \tag{2.7}
$$

$$
\mathbf{X}^2 = X_1^2+X_2^2+X_3^2, \tag{2.8}
$$

$$
v^i = (-\Omega X_2,\ \Omega X_1,\ 0). \tag{2.9}
$$

In the following calculation, we integrate only over star 1 because the total mass quadrupole moment is twice as large as that of star 1. Moreover, as shown in Ref. [10], the deformation from the Newtonian ellipsoid becomes higher order effect of R^{-1}. Therefore, we integrate over the ellipsoid simply.

1. Newtonian order

The quadrupole moment of the Newtonian order appears in the first term of Eq. (2.4) and expressed as

$$
D_{ij} \equiv \int d^3X\rho\biggl(X_iX_j - \frac{1}{3}\delta_{ij}\mathbf{X}^2\biggr). \tag{2.10}
$$

Using Eq. (2.1), we get the components as

$$
D_{11} = \hat{D}\cos^2\Omega t + \text{const.}, \tag{2.11}
$$

$$
D_{22} = -\hat{D}\sin^2\Omega t + \text{const.}, \tag{2.12}
$$

$$
D_{12} = \hat{D}\sin\Omega t\cos\Omega t, \tag{2.13}
$$

where $\hat{D} = I_{11} - I_{22} + R^2M/4$. In these equations, we use $M \equiv \int\rho d^3x$ as the Newtonian mass. Since D_{33} does not depend on time and the other terms vanish, we do not write them here. Note that in these components, only the contribution from star 1 is included.

2. 1PN order

Next, we calculate the 1PN order terms. For the 1PN order terms in Eq. (2.4), we define

$$
V_{ij} \equiv 2\int d^3X\rho v^2\hat{X}_{ij}, \tag{2.14}
$$

$$
U_{ij}^{1\to1} \equiv 2\int d^3X\rho U^{1\to1}\hat{X}_{ij}, \tag{2.15}
$$

$$
U_{ij}^{2\to1} \equiv 2\int d^3X\rho U^{2\to1}\hat{X}_{ij}, \tag{2.16}
$$

$$
P_{ij} \equiv 3\int d^3X\rho\frac{P}{\rho}\hat{X}_{ij}, \tag{2.17}
$$

$$
Q_{ij} \equiv \frac{1}{14}\frac{d^2}{dt^2}\biggl(\int d^3X\rho\hat{X}_{ij}\mathbf{X}^2\biggr), \tag{2.18}
$$

$$
R_{ij} \equiv -\frac{20}{21}\frac{d}{dt}\biggl(\int d^3X\rho v_k\hat{X}_{ijk}\biggr), \tag{2.19}
$$

because it is convenient to calculate the terms separately. In these equations, we use

$$U^{1\to1} = \pi\rho\Big(A_0 - \sum_l A_l x_l^2\Big), \tag{2.20}$$

$$\begin{aligned} U^{2\to1} &= \frac{M}{R}\left[1 - \frac{x_1}{R} + \frac{2x_1^2 - x_2^2 - x_3^2}{2R^2} + \frac{-2x_1^3 + 3x_1(x_2^2 + x_3^2)}{2R^3} + O(R^{-4})\right] \\ &\quad + \frac{3\rlap{\,-}I_{11}}{2R^3}\left(1 - \frac{3x_1}{R} + O(R^{-2})\right) + O(R^{-5}), \end{aligned} \tag{2.21}$$

$$P = P_0\Big(1 - \sum_l \frac{x_l^2}{a_l^2}\Big), \tag{2.22}$$

$$P_0 = \frac{\rho}{3}\left[\pi\rho A_0 - \frac{\Omega_{\rm N}^2}{2}(a_1^2 + a_2^2) - \frac{M}{2R^3}(2a_1^2 - a_2^2 - a_3^2)\right] + O(R^{-5}), \tag{2.23}$$

where $U^{1\to1}$, $U^{2\to1}$ and P denote the Newtonian potential generated by star 1 itself, the Newtonian potential generated by star 2, and the pressure, respectively [10]. $A_{ij\cdots}$ are index symbols introduced by Chandrasekhar [20] and $A_0 = \sum_l A_l a_l^2$ is calculated from [20]

$$A_0 = a_1 a_2 a_3 \int_0^\infty \frac{du}{\sqrt{(a_1^2 + u)(a_2^2 + u)(a_3^2 + u)}}, \tag{2.24}$$

$$= a_1^2 \alpha_2 \alpha_3 \int_0^\infty \frac{dt}{\sqrt{(1 + t)(\alpha_2^2 + t)(\alpha_3^2 + t)}} \equiv a_1^2 \tilde{A}_0, \tag{2.25}$$

where $\alpha_2 = a_2/a_1$ and $\alpha_3 = a_3/a_1$ are axial ratios.

When we derive the 1PN order terms, we need to calculate only some combinations: The 1PN order terms in $M_{ij}^{(3)} M_{ij}^{(3)}$ are given by

$$\begin{aligned} & D_{ij}^{(3)}\Big(V_{ij}^{(3)} + U_{ij}^{1\to1(3)} + U_{ij}^{2\to1(3)} + P_{ij}^{(3)} + Q_{ij}^{(3)} + R_{ij}^{(3)}\Big) \\ &= D_{11}^{(3)}\Big[V_{11}^{(3)} - V_{22}^{(3)} + U_{11}^{1\to1(3)} - U_{22}^{1\to1(3)} + U_{11}^{2\to1(3)} - U_{22}^{2\to1(3)} + P_{11}^{(3)} - P_{22}^{(3)} + Q_{11}^{(3)} - Q_{22}^{(3)} + R_{11}^{(3)} - R_{22}^{(3)}\Big] \\ &\quad + 2D_{12}^{(3)}\Big[V_{12}^{(3)} + U_{12}^{1\to1(3)} + U_{12}^{2\to1(3)} + P_{12}^{(3)} + Q_{12}^{(3)} + R_{12}^{(3)}\Big], \end{aligned} \tag{2.26}$$

where we use the relation $D_{22}^{(i)} = -D_{11}^{(i)}$. Then, we need to calculate only two combinations of the components expressed as $[(1,1)-(2,2)]$ and $[(1,2)]$. We separately show the terms of the 1PN order in Eq. (2.26) in the following.

(1.1) $V_{11} - V_{22}$:

$$\begin{aligned} V_{11} - V_{22} &= 2\Omega^2 \int d^3X \rho (X_1^2 + X_2^2)(X_1^2 - X_2^2) \\ &= 2\Omega^2 \hat{V} \cos 2\Omega t, \end{aligned} \tag{2.27}$$

where

$$\hat{V} = \frac{R^4 M}{16} + \frac{3R^2}{2} I_{11} + I_{1111} - I_{2222}, \tag{2.28}$$

and

$$\begin{aligned} I_{ijkl} &\equiv \int d^3x \rho x_i x_j x_k x_l, \\ &= \frac{M}{35} \times \left\{ \begin{array}{ll} 3a_i^4 & (i = j = k = l) \\ a_i^2 a_k^2 & (i = j \neq k = l). \end{array} \right. \end{aligned} \tag{2.29}$$

(1.2) V_{12}:

$$
\begin{aligned}
V_{12} &= 2\Omega^2 \int d^3X \rho (X_1^2 + X_2^2) X_1 X_2 \\
&= \Omega^2 \hat{V} \sin 2\Omega t.
\end{aligned} \tag{2.30}
$$

(2.1) $U_{11}^{1\to 1} - U_{22}^{1\to 1}$:

$$
\begin{aligned}
U_{11}^{1\to 1} - U_{22}^{1\to 1} &= 2 \int d^3X \rho U^{1\to 1} (X_1^2 - X_2^2) \\
&= 2\hat{U}^{1\to 1} \cos 2\Omega t,
\end{aligned} \tag{2.31}
$$

where

$$
\hat{U}^{1\to 1} = \pi\rho \left[A_0 \left(I_{11} - I_{22} + \frac{R^2 M}{4} \right) - \sum_l A_l \left(I_{11ll} - I_{22ll} + \frac{R^2}{4} I_{ll} \right) \right]. \tag{2.32}
$$

(2.2) $U_{12}^{1\to 1}$:

$$
\begin{aligned}
U_{12}^{1\to 1} &= 2 \int d^3X \rho U^{1\to 1} X_1 X_2 \\
&= \hat{U}^{1\to 1} \sin 2\Omega t.
\end{aligned} \tag{2.33}
$$

(3.1) $U_{11}^{2\to 1} - U_{22}^{2\to 1}$:

$$
\begin{aligned}
U_{11}^{2\to 1} - U_{22}^{2\to 1} &= 2 \int d^3X \rho U^{2\to 1} (X_1^2 - X_2^2) \\
&= 2\hat{U}^{2\to 1} \cos 2\Omega t + O(R^{-3}),
\end{aligned} \tag{2.34}
$$

where

$$
\hat{U}^{2\to 1} = \frac{M}{R} \left(\frac{R^2 M}{4} - I_{22} + \frac{3}{4} \not{I}_{11} \right). \tag{2.35}
$$

(3.2) $U_{12}^{2\to 1}$:

$$
\begin{aligned}
U_{12}^{2\to 1} &= 2 \int d^3X \rho U^{2\to 1} X_1 X_2 \\
&= \hat{U}^{2\to 1} \sin 2\Omega t + O(R^{-3}).
\end{aligned} \tag{2.36}
$$

(4.1) $P_{11} - P_{22}$:

$$
\begin{aligned}
P_{11} - P_{22} &= 3 \int d^3X \rho \frac{P}{\rho} (X_1^2 - X_2^2) \\
&= 3\hat{P} \cos 2\Omega t,
\end{aligned} \tag{2.37}
$$

where

$$
\hat{P} = \frac{P_0}{\rho} \left[I_{11} - I_{22} + \frac{R^2 M}{4} - \sum_l \frac{1}{a_l^2} \left(I_{11ll} - I_{22ll} + \frac{R^2}{4} I_{ll} \right) \right]. \tag{2.38}
$$

(4.2) P_{12}:

$$P_{12} = 3 \int d^3X \rho \frac{P}{\rho} X_1 X_2$$
$$= \frac{3}{2} \hat{P} \sin 2\Omega t. \tag{2.39}$$

(5.1) $Q_{11} - Q_{22}$:

$$Q_{11} - Q_{22} = \frac{1}{14} \frac{d^2}{dt^2} \left(\int d^3X \rho (X_1^2 - X_2^2) \mathbf{X}^2 \right)$$
$$= -\frac{2}{7} \Omega^2 \hat{Q} \cos 2\Omega t, \tag{2.40}$$

where

$$\hat{Q} = \frac{R^4 M}{16} + \frac{R^2}{4}(6I_{11} + I_{33}) + I_{1111} - I_{2222} + I_{1133} - I_{2233}. \tag{2.41}$$

(5.2) Q_{12}:

$$Q_{12} = \frac{1}{14} \frac{d^2}{dt^2} \left(\int d^3X \rho X_1 X_2 \mathbf{X}^2 \right)$$
$$= -\frac{1}{7} \Omega^2 \hat{Q} \sin 2\Omega t. \tag{2.42}$$

(6.1) $R_{11} - R_{22}$:

$$R_{11} - R_{22} = -\frac{16}{21} \Omega \frac{d}{dt} \left(\int d^3X \rho X_1 X_2 \mathbf{X}^2 \right)$$
$$= -\frac{16}{21} \Omega^2 \hat{Q} \cos 2\Omega t. \tag{2.43}$$

(6.2) R_{12}:

$$R_{12} = \frac{4}{21} \Omega \frac{d}{dt} \left(\int d^3X \rho (X_1^2 - X_2^2) \mathbf{X}^2 \right)$$
$$= -\frac{8}{21} \Omega^2 \hat{Q} \sin 2\Omega t. \tag{2.44}$$

B. Mass octupole moment and current quadrupole moment

In this subsection, we discuss on the mass octupole moment M_{ijk} and the current quadrupole one S_{ij}. They are written as

$$M_{ijk}(t) = \int d^3X \rho \hat{X}_{ijk}, \tag{2.45}$$
$$S_{ij}(t) = \int d^3X \rho \epsilon_{kl<i} \hat{X}_{j>k} v_l, \tag{2.46}$$

where

$$A_{<i}B_{j>} = \frac{1}{2}(A_i B_j + B_i A_j) - \frac{1}{3}\delta_{ij}(A_k B_k). \tag{2.47}$$

M_{ijk} and S_{ij} are the odd terms of the coordinates X_1, X_2, and X_3. Therefore, the contribution from star 2 has the opposite sign of that of star 1. Then, the total M_{ijk} and S_{ij} vanish as

$$(M_{ijk})_{\rm tot} = {}^1M_{ijk} + {}^2M_{ijk} = 0, \tag{2.48}$$

$$(S_{ij})_{\rm tot} = {}^1S_{ij} + {}^2S_{ij} = 0, \tag{2.49}$$

where in the right hand side, superscripts 1 and 2 denote the contributions from star 1 and star 2, respectively. Therefore, they have no contributions to the energy and the angular momentum loss rates in the case of the identical star binary.

III. THE ENERGY AND ANGULAR MOMENTUM LOSS RATES

Our purpose is to know the finite size effects of the 1PN order in gravitational radiation. In the following, we calculate the energy and angular momentum loss rates up to $O(R^{-3})$ in the 1PN order beyond the Newtonian order of the quadrupole formula.

A. The energy loss rate

Taking into account the fact stated in subsection II B, the energy loss rate is expressed as

$$\begin{aligned}\frac{dE}{dt} &= -\frac{1}{5c^5}\left[(M_{ij}^{(3)})_{\rm tot}(M_{ij}^{(3)})_{\rm tot}\right] \\ &= -\frac{1}{5c^5}\left[(2D_{ij}^{(3)})(2D_{ij}^{(3)}) + \frac{2}{c^2}(2D_{ij}^{(3)})\left\{2V_{ij}^{(3)} + 2U_{ij}^{1\to1(3)} + 2U_{ij}^{2\to1(3)} + 2P_{ij}^{(3)} + 2Q_{ij}^{(3)} + 2R_{ij}^{(3)}\right\}\right].\end{aligned} \tag{3.1}$$

The contribution from the Newtonian order is

$$(2D_{ij}^{(3)})(2D_{ij}^{(3)}) = 128\Omega^6\hat{D}^2. \tag{3.2}$$

The contributions from the 1PN order are written as follows:

$$8D_{ij}^{(3)}V_{ij}^{(3)} = 512\Omega^8\hat{D}\hat{V}, \tag{3.3}$$

$$8D_{ij}^{(3)}U_{ij}^{1\to1(3)} = 512\Omega^6\hat{D}\hat{U}^{1\to1}, \tag{3.4}$$

$$8D_{ij}^{(3)}U_{ij}^{2\to1(3)} = 512\Omega^6\hat{D}\hat{U}^{2\to1}, \tag{3.5}$$

$$8D_{ij}^{(3)}P_{ij}^{(3)} = 768\Omega^6\hat{D}\hat{P}, \tag{3.6}$$

$$8D_{ij}^{(3)}Q_{ij}^{(3)} = -\frac{512}{7}\Omega^8\hat{D}\hat{Q}, \tag{3.7}$$

$$8D_{ij}^{(3)}R_{ij}^{(3)} = -\frac{4096}{21}\Omega^8\hat{D}\hat{Q}. \tag{3.8}$$

Collecting the results we derived above, we can get the energy loss rate as

$$\begin{aligned}\frac{dE}{dt} = -\frac{64M^5}{5c^5R^5}&\left[1 + \frac{1}{5R^2}\left\{8(a_1^2 - a_2^2) + 9(2a_1^2 - a_2^2 - a_3^2)\right\} + O(R^{-4})\right. \\ &+\frac{1}{c^2}\left[10\pi\rho A_0 - \frac{151M}{84R} + \frac{4\pi\rho A_0}{5R^2}\left\{\frac{412}{21}(a_1^2 - a_2^2) + \frac{111}{5}(2a_1^2 - a_2^2 - a_3^2)\right\}\right. \\ &\left.\left.-\frac{M}{210R^3}(1658a_1^2 - 1103a_2^2 - 16a_3^2) + O(R^{-4})\right]\right],\end{aligned} \tag{3.9}$$

where we use the angular velocity of the 1PN order [10]

$$\Omega^2 = \frac{2M}{R^3}\left[1 + \frac{1}{c^2}\left\{2\pi\rho A_0 - \frac{9M}{4R} - \frac{M}{10R^3}(28a_1^2 - 14a_2^2 - 9a_3^2) + O(R^{-4})\right\}\right]$$
$$+\frac{18\not{I}_{11}}{R^5}\left(1 + \frac{28}{15c^2}\pi\rho A_0 + O(R^{-2})\right). \tag{3.10}$$

To express Eq. (3.9), we have used M, but it does not conserve through the sequence of the binary. Moreover, we simply used R as the orbital separation, but in the 1PN case, the center of mass deviates from that in the Newtonian one and as a result, the orbital separation also deviates from R. We need a conserved mass and an appropriate definition of the center of mass.

If we use the conserved *proper* mass

$$M_* = \int d^3x\rho\left[1 + \frac{1}{c^2}\left(\frac{v^2}{2} + 3U\right)\right]$$
$$= M\left[1 + \frac{1}{c^2}\left\{\frac{12\pi\rho A_0}{5} + \frac{13M}{4R} + \frac{M}{20R^3}(34a_1^2 - 11a_2^2 - 15a_3^2) + O(R^{-5})\right\}\right], \tag{3.11}$$

and define the center of mass by it as

$$x_*^i = \frac{1}{M_*}\int d^3x\rho_* x^i, \tag{3.12}$$

where

$$\rho_* \equiv \rho\left[1 + \frac{1}{c^2}\left(\frac{v^2}{2} + 3U\right)\right], \tag{3.13}$$

the orbital separation should be replaced by

$$R_* = R\left[1 + \frac{1}{c^2}\left\{-\frac{4Ma_1^2}{5R^3} + O(R^{-5})\right\}\right]. \tag{3.14}$$

Using these expressions, the energy loss rate is rewritten as

$$\frac{dE}{dt} = -\frac{64M_*^5}{5c^5R_*^5}\left[1 + \frac{1}{5R_*^2}\left\{8(a_{1*}^2 - a_{2*}^2) + 9(2a_{1*}^2 - a_{2*}^2 - a_{3*}^2)\right\} + O(R_*^{-4})\right.$$
$$+\frac{1}{c^2}\left[-2\pi\rho A_{0*} - \frac{379M_*}{21R_*} - \frac{8\pi\rho A_{0*}}{25R_*^2}\left\{\frac{398}{21}(a_{1*}^2 - a_{2*}^2) + 21(2a_{1*}^2 - a_{2*}^2 - a_{3*}^2)\right\}\right.$$
$$\left.\left.-\frac{M_*}{210R_*^3}(24394a_{1*}^2 - 14830a_{2*}^2 - 7765a_{3*}^2) + O(R_*^{-4})\right]\right], \tag{3.15}$$

where we use the relation

$$a_i^2 = a_{i*}^2\left[1 - \frac{1}{c^2}\left\{\frac{8\pi\rho A_{0*}}{5} + \frac{13M_*}{6R} + \frac{M_*}{30R^3}(34a_{1*}^2 - 11a_{2*}^2 - 15a_{3*}^2)\right\}\right], \tag{3.16}$$

and $a_{1*}a_{2*}a_{3*} = M_*/(4\pi\rho/3) =$ constant in the 1PN order. The expression of Eq. (3.15) seems to depend on the internal structure of the star of binary even if we take the limits $a_i/R \to 0$. Thus, M_* is not appropriate in comparing the energy luminosity with that for the point particle case. This is simply because M_* is not the gravitational mass. Instead of M_*, we need an appropriate *gravitational* mass.

If we use the parameterized post-Newtonian (PPN) mass [21]

$$M_{\rm PPN} = \int d^3x\rho\left[1 + \frac{1}{c^2}\left(\frac{v^2}{2} + 3U - \frac{1}{2}U_{\rm self} + \frac{v_{\rm self}^2}{2}\right)\right]$$
$$= M\left[1 + \frac{1}{c^2}\left\{2\pi\rho A_0 + \frac{13M}{4R} + \frac{M}{20R^3}(38a_1^2 - 7a_2^2 - 15a_3^2) + O(R^{-5})\right\}\right], \tag{3.17}$$

with the center of mass of each star defined as

$$x^i_{\rm PPN} = \frac{1}{M_{\rm PPN}} \int d^3x\rho x^i \left[1 + \frac{1}{c^2}\left(\frac{v^2}{2} + 3U - \frac{1}{2}U_{\rm self} + \frac{v^2_{\rm self}}{2}\right)\right] \tag{3.18}$$

and the orbital separation as

$$R_{\rm PPN} = R\left[1 + \frac{1}{c^2}\left\{-\frac{4Ma_1^2}{5R^3} + O(R^{-5})\right\}\right], \tag{3.19}$$

the leading internal structure dependent term in the 1PN order is renormalized. Substituting the PPN mass and $R_{\rm PPN}$ into Eq. (3.9), we rewrite the energy loss rate as

$$\begin{aligned}\frac{dE}{dt} = &-\frac{64M^5_{\rm PPN}}{5c^5R^5_{\rm PPN}}\Bigg[1 + \frac{1}{5R^2_{\rm PPN}}\left\{8(a^2_{\rm 1PPN} - a^2_{\rm 2PPN}) + 9(2a^2_{\rm 1PPN} - a^2_{\rm 2PPN} - a^2_{\rm 3PPN})\right\} + O(R^{-4}_{\rm PPN}) \\ &+\frac{1}{c^2}\Bigg[-\frac{379M_{\rm PPN}}{21R_{\rm PPN}} - \frac{2\pi\rho A_{\rm 0PPN}}{15R^2_{\rm PPN}}\left\{\frac{128}{7}(a^2_{\rm 1PPN} - a^2_{\rm 2PPN}) + \frac{99}{5}(2a^2_{\rm 1PPN} - a^2_{\rm 2PPN} - a^2_{\rm 3PPN})\right\} \\ &-\frac{M_{\rm PPN}}{210R^3_{\rm PPN}}(24604a^2_{\rm 1PPN} - 14620a^2_{\rm 2PPN} - 7765a^2_{\rm 3PPN}) + O(R^{-4}_{\rm PPN})\Bigg]\Bigg],\end{aligned} \tag{3.20}$$

where we use the relation

$$a_i^2 = a^2_{i\rm PPN}\left[1 - \frac{1}{c^2}\left\{\frac{4\pi\rho A_{\rm 0PPN}}{3} + \frac{13M_{\rm PPN}}{6R} + \frac{M_{\rm PPN}}{30R^3}(38a^2_{\rm 1PPN} - 7a^2_{\rm 2PPN} - 15a^2_{\rm 3PPN})\right\}\right]. \tag{3.21}$$

Then we recover the energy loss rate given for the point equal mass binary case [6] when we take the limits $a_i/R \to 0$. (However, $M_{\rm PPN}$ appears to be the imperfect definition of gravitational mass as shown below.)

From Eq. (3.20), we find that the leading finite size effect of 1PN order is only of $O(\rho A_0)(\sim O(M/a_i))$ smaller than the Newtonian finite size term. The next order terms are of $O(M/R)$ smaller than the Newtonian term. Here, we note that the effect of the spin-orbit (SO) coupling appears in the same order as the latter terms for the case of the corotating binary case. Including the SO coupling term, the energy loss rate of the point particle binary of equal mass is written as [22]

$$\frac{dE}{dt} = -\frac{64M^5_{\rm PPN}}{5c^5R^5_{\rm PPN}}\left[1 - \frac{1}{c^2}\left\{\frac{379M_{\rm PPN}}{21R_{\rm PPN}} + \frac{74M_{\rm PPN}}{15R^3_{\rm PPN}}(a^2_{\rm 1PPN} + a^2_{\rm 2PPN})\right\}\right]. \tag{3.22}$$

We can see from this equation that the latter finite size terms of the 1PN order in Eq. (3.20) is not explained only by the SO coupling term. In order to examine whether this fact is really true or not, we rewrite dE/dt using the angular velocity which is the invariant value of the coordinate.

Using the angular velocity, the orbital separation is written as

$$\begin{aligned}R = &\left(\frac{2M_{\rm PPN}}{\Omega^2}\right)^{1/3}\Bigg[1 + \frac{1}{5}\left(\frac{\Omega^2}{2M_{\rm PPN}}\right)^{2/3}(2a^2_{\rm 1PPN} - a^2_{\rm 2PPN} - a^2_{\rm 3PPN}) \\ &+\frac{1}{c^2}\Bigg\{-\frac{11M_{\rm PPN}}{6}\left(\frac{\Omega^2}{2M_{\rm PPN}}\right)^{1/3} - \frac{22\pi\rho A_{\rm 0PPN}}{75}\left(\frac{\Omega^2}{2M_{\rm PPN}}\right)^{2/3}(2a^2_{\rm 1PPN} - a^2_{\rm 2PPN} - a^2_{\rm 3PPN}) \\ &-\frac{M_{\rm PPN}}{30}\left(\frac{\Omega^2}{2M_{\rm PPN}}\right)(2a^2_{\rm 1PPN} + 5a^2_{\rm 2PPN} + 6a^2_{\rm 3PPN})\Bigg\}\Bigg].\end{aligned} \tag{3.23}$$

Also if we use M_*, the orbital separation is written as

$$\begin{aligned}R = &\left(\frac{2M_*}{\Omega^2}\right)^{1/3}\Bigg[1 + \frac{1}{5}\left(\frac{\Omega^2}{2M_*}\right)^{2/3}(2a^2_{1*} - a^2_{2*} - a^2_{3*}) + \frac{1}{c^2}\Bigg\{-\frac{2\pi\rho A_{0*}}{15} - \frac{11M_*}{6}\left(\frac{\Omega^2}{2M_*}\right)^{1/3} \\ &-\frac{8\pi\rho A_{0*}}{25}\left(\frac{\Omega^2}{2M_*}\right)^{2/3}(2a^2_{1*} - a^2_{2*} - a^2_{3*}) - \frac{M_*}{10}\left(\frac{\Omega^2}{2M_*}\right)(a^2_{2*} + 2a^2_{3*})\Bigg\}\Bigg].\end{aligned} \tag{3.24}$$

Substituting these expressions into the equation of the energy loss rate, we have

$$
\begin{aligned}
\frac{dE}{dt} = -\frac{2}{5c^5}(2M_{\rm PPN}\Omega)^{10/3}\Biggl[1 &+ \frac{(2M_{\rm PPN}\Omega)^{4/3}}{5M_{\rm PPN}^2}\Biggl\{2(a_{1\rm PPN}^2 - a_{2\rm PPN}^2) + 2a_{1\rm PPN}^2 - a_{2\rm PPN}^2 - a_{3\rm PPN}^2\Biggr\} \\
&+\frac{1}{c^2}\Biggl[-\frac{373}{84}(2M_{\rm PPN}\Omega)^{2/3} \\
&\qquad -\frac{2\pi\rho A_{0\rm PPN}}{525M_{\rm PPN}^2}(2M_{\rm PPN}\Omega)^{4/3}\Biggl\{160(a_{1\rm PPN}^2 - a_{2\rm PPN}^2) + 77(2a_{1\rm PPN}^2 - a_{2\rm PPN}^2 - a_{3\rm PPN}^2)\Biggr\} \\
&\qquad -\frac{(2M_{\rm PPN}\Omega)^2}{840M_{\rm PPN}^2}(2602a_{1\rm PPN}^2 - 1697a_{2\rm PPN}^2 - 443a_{3\rm PPN}^2)\Biggr]\Biggr],
\end{aligned}
\tag{3.25}
$$

or

$$
\begin{aligned}
\frac{dE}{dt} = -\frac{2}{5c^5}(2M_*\Omega)^{10/3}\Biggl[1 &+ \frac{(2M_*\Omega)^{4/3}}{5M_*^2}\Biggl\{2(a_{1*}^2 - a_{2*}^2) + 2a_{1*}^2 - a_{2*}^2 - a_{3*}^2\Biggr\} \\
&+\frac{1}{c^2}\Biggl[-\frac{4\pi\rho A_{0*}}{3} - \frac{373}{84}(2M_*\Omega)^{2/3} - \frac{2\pi\rho A_{0*}}{175M_*^2}(2M_*\Omega)^{4/3}\Biggl\{100(a_{1*}^2 - a_{2*}^2) + 49(2a_{1*}^2 - a_{2*}^2 - a_{3*}^2)\Biggr\} \\
&\qquad -\frac{(2M_*\Omega)^2}{840M_*^2}(2532a_{1*}^2 - 1767a_{2*}^2 - 443a_{3*}^2)\Biggr]\Biggr].
\end{aligned}
\tag{3.26}
$$

On the other hand, in the point particle case, the energy loss rate is written as

$$
\frac{dE}{dt} = -\frac{2}{5c^5}(2M_{\rm PPN}\Omega)^{10/3}\Biggl[1 + \frac{1}{c^2}\Biggl\{-\frac{373}{84}(2M_{\rm PPN}\Omega)^{2/3} - \frac{(2M_{\rm PPN}\Omega)^2}{5M_{\rm PPN}^2}(a_{1\rm PPN}^2 + a_{2\rm PPN}^2)\Biggr\}\Biggr]. \tag{3.27}
$$

Comparing the last term of Eq. (3.25) with that of Eq. (3.27), we find that they do not agree. A part of the reason seems due to the fact that the 1PN quadrupole dependent term really appears in the last terms of Eq. (3.25). However, we cannot still explain the disagreement completely because there remains terms except for the SO coupling one even if we take a spherical limit $a_{1\rm PPN} = a_{2\rm PPN} = a_{3\rm PPN}$. Then, what is the reason why the terms remain? We guess that this is due to the imperfection of the definition of gravitational mass. We adopt the PPN mass as gravitational mass of each star to write the equation of the energy loss rate, but the PPN mass can renormalize only the self-gravity and spin into mass. In the context of this paper, we need a conserved gravitational mass which renormalizes the quadrupole term produced by the tidal force of the companion star. As far as we know, however, no one has proposed such a mass. Nevertheless, $M_{\rm PPN}$ is not expected to be different from "true gravitational mass" $M_{\rm true}$ so much because

$$
M_{\rm true} = M_{\rm PPN}\left[1 + O\left(\frac{Ma_i^2}{c^2R^3}\right)\right]. \tag{3.28}
$$

Hence, we regard $M_{\rm PPN}$ as an approximate gravitational mass in the following.

B. The angular momentum loss rate

Next, we calculate the angular momentum loss rate. As shown by Ostriker and Gunn [23], the angular momentum loss rate has the relation

$$
\frac{dJ^3}{dt} = \frac{1}{\Omega}\frac{dE}{dt} \tag{3.29}
$$

for any stable, uniformly rotating, equilibrium configuration. Then, the angular momentum loss rate is written as

$$
\begin{aligned}
\frac{dJ^3}{dt} = -\frac{4M_{\rm PPN}}{5c^5}(2M_{\rm PPN}\Omega)^{7/3}\Biggl[1 &+ \frac{(2M_{\rm PPN}\Omega)^{4/3}}{5M_{\rm PPN}^2}\Biggl\{2(a_{1\rm PPN}^2 - a_{2\rm PPN}^2) + 2a_{1\rm PPN}^2 - a_{2\rm PPN}^2 - a_{3\rm PPN}^2\Biggr\} \\
&+\frac{1}{c^2}\Biggl[-\frac{373}{84}(2M_{\rm PPN}\Omega)^{2/3}
\end{aligned}
$$

$$-\frac{2\pi\rho A_{0\mathrm{PPN}}}{525M_{\mathrm{PPN}}^2}(2M_{\mathrm{PPN}}\Omega)^{4/3}\left\{160(a_{1\mathrm{PPN}}^2-a_{2\mathrm{PPN}}^2)+77(2a_{1\mathrm{PPN}}^2-a_{2\mathrm{PPN}}^2-a_{3\mathrm{PPN}}^2)\right\}$$
$$-\frac{(2M_{\mathrm{PPN}}\Omega)^2}{840M_{\mathrm{PPN}}^2}(2602a_{1\mathrm{PPN}}^2-1697a_{2\mathrm{PPN}}^2-443a_{3\mathrm{PPN}}^2)\Bigg]\Bigg], \tag{3.30}$$

or

$$\frac{dJ^3}{dt}=-\frac{4M_*}{5c^5}(2M_*\Omega)^{7/3}\Bigg[1+\frac{(2M_*\Omega)^{4/3}}{5M_*^2}\left\{2(a_{1*}^2-a_{2*}^2)+2a_{1*}^2-a_{2*}^2-a_{3*}^2\right\}$$
$$+\frac{1}{c^2}\Bigg[-\frac{4\pi\rho A_{0*}}{3}-\frac{373}{84}(2M_*\Omega)^{2/3}-\frac{2\pi\rho A_{0*}}{175M_*^2}(2M_*\Omega)^{4/3}\left\{100(a_{1*}^2-a_{2*}^2)+49(2a_{1*}^2-a_{2*}^2-a_{3*}^2)\right\}$$
$$-\frac{(2M_*\Omega)^2}{840M_*^2}(2532a_{1*}^2-1767a_{2*}^2-443a_{3*}^2)\Bigg]\Bigg]. \tag{3.31}$$

We will also show the direct calculation in Appendix A.

IV. NUMERICAL RESULTS

In this section, we calculate the energy loss rate along the equilibrium sequence of the binary. The method is as follows.

(i) Using the equilibrium equations for a corotating binary, we construct the sequence of the binary. The equations are written as

$$-\frac{P_0}{\rho}=-\pi\rho a_1^2A_1+\frac{M}{R^3}a_1^2+\frac{\Omega_{\mathrm{N}}^2}{2}a_1^2, \tag{4.1}$$
$$-\frac{P_0}{\rho}=-\pi\rho a_2^2A_2-\frac{M}{2R^3}a_2^2+\frac{\Omega_{\mathrm{N}}^2}{2}a_2^2, \tag{4.2}$$
$$-\frac{P_0}{\rho}=-\pi\rho a_3^2A_3-\frac{M}{2R^3}a_3^2, \tag{4.3}$$

where Ω_{N} denotes the Newtonian angular velocity written by

$$\Omega_{\mathrm{N}}^2=\frac{2M}{R^3}+\frac{18I\!\!\!/_{11}}{R^5}. \tag{4.4}$$

At this stage, we determine $\tilde{R}=R/a_1$, $\alpha_2=a_2/a_1$ and $\alpha_3=a_3/a_1$.

(ii) Inserting the equilibrium figures into the equation of the orbital angular velocity of 1PN order

$$\Omega=\left(\frac{2M_*}{R^3}\right)^{1/2}\Bigg[1+\frac{3}{10R^2}(2a_{1*}^2-a_{2*}^2-a_{3*}^2)-\frac{1}{c^2}\Bigg\{\frac{\pi\rho A_{0*}}{5}+\frac{11M_*}{4R}$$
$$+\frac{29\pi\rho A_{0*}}{50R^2}(2a_{1*}^2-a_{2*}^2-a_{3*}^2)+\frac{M_*}{40R^3}(154a_{1*}^2-71a_{2*}^2-65a_{3*}^2)\Bigg\}\Bigg] \tag{4.5}$$

and the equation of the total energy of the binary [10], we determine the initial and final points. Here, we regard the point which satisfies the condition $\Omega/\pi = 10$Hz as the initial one and the point which satisfies $\Omega/\pi = 1000$Hz or reaches the energy minimum (ISCO)[‡] as the final one.

(iii) Substituting the equilibrium figures into Eq. (3.25), we are able to have the energy loss rate. However, Eq. (3.25) is not written by conserved quantities. Then, we express Eq. (3.25) as

$$\left|\bar{\dot{E}}\right|=1+\bar{\dot{E}}_{\mathrm{N-finite}}+\bar{\dot{E}}_{\mathrm{1PN}}+\bar{\dot{E}}_{\mathrm{1PN-finite}} \tag{4.6}$$

[‡]In the corotating binary case, we call it the innermost stable corotating circular orbit (ISCCO) [10].

where

$$\bar{\dot{E}} \equiv \frac{dE}{dt} \bigg/ \left(\frac{dE}{dt}\right)_{\mathrm{N}}, \tag{4.7}$$

$$\left(\frac{dE}{dt}\right)_{\mathrm{N}} = -\frac{2}{5c^5}(2M_{\mathrm{PPN}}\Omega)^{10/3}, \tag{4.8}$$

$$\bar{\dot{E}}_{\mathrm{N-finite}} = \frac{1}{5C_{\mathrm{s}}^2}\left(\frac{2M_*}{c^2}\frac{\Omega}{c}\right)^{4/3}\frac{1}{(\alpha_2\alpha_3)^{2/3}}\Big\{2(1-\alpha_2^2)+2-\alpha_2^2-\alpha_3^2\Big\}, \tag{4.9}$$

$$\bar{\dot{E}}_{\mathrm{1PN}} = -\frac{373}{84}\left(\frac{2M_*}{c^2}\frac{\Omega}{c}\right)^{2/3}, \tag{4.10}$$

$$\bar{\dot{E}}_{\mathrm{1PN-finite}} = -\frac{1}{350C_{\mathrm{s}}}\left(\frac{2M_*}{c^2}\frac{\Omega}{c}\right)^{4/3}\frac{\tilde{A}_0}{(\alpha_2\alpha_3)^{4/3}}\Big\{160(1-\alpha_2^2)+77(2-\alpha_2^2-\alpha_3^2)\Big\}. \tag{4.11}$$

In these equations, we use the compactness parameter

$$C_{\mathrm{s}} \equiv \frac{M_*}{c^2 a_*}. \tag{4.12}$$

In calculation of the energy loss rate, we neglect the last term of Eq. (3.25), because this term includes the ambiguity of the definition of mass and we cannot separate the SO coupling term from it as mentioned in the last of subsection III A.

We repeat this procedure changing the compactness of the star M_*/c^2a_* and the conserved proper mass M_*.

Figures 1(a)–3(c) show the results we have using this procedure. It is found from these figures that the finite size effect of the 1PN order is only a few factor less than that of the Newtonian order. We can explain this feature by comparing Eq. (4.9) with Eq. (4.11). Essentially, the difference between these equations is $O(C_{\mathrm{s}})$. Therefore, the finite size effect of the 1PN order becomes $O(C_{\mathrm{s}})$ less than that of the Newtonian order.

There is another feature to mention specially. Comparing Eq. (4.9) with Eq. (4.11), it is found that the contribution from the 1PN order has the opposite sign of that from the Newtonian order. We explain this fact as follows: Including the 1PN order terms, the self-gravity of each star of binary becomes stronger than that in the Newtonian gravity. Then, the stars become more compact, and it is more difficult to deform them. This leads to the decrease of the finite size effect. Moreover, it is found from these figures that when we increase the compactness parameter fixing M_*, both the finite size effects of the Newtonian order and the 1PN one decrease, and also the difference between their absolute values decreases. The explanation for these behaviors is the same as the above one, i.e., the self-gravity of each star becomes stronger.

Finally, we discuss on the inclinations of the lines in the figures. First, the inclination of the 1PN line becomes 2/3 in the log-log plot figures because of Eq. (4.10). Next, we can see from the figures that the inclination of the finite size effect lines is ~ 3.4. This is because the deviation from the spherically symmetric star is made by the tidal force $\sim M/R^3$. We find from Eq. (3.10) that M/R^3 is equal to Ω^2. Then, combining this fact with Eq. (4.9) or (4.11), we can conclude the inclination of the finite size effect lines becomes $\sim 10/3$.

In Table I, we show the results of the initial and final orbits. Note that the frequencies of the final orbits are 1000Hz in the cases of $(M_*/M_\odot,\ M_*/c^2a_*) = (1.4,\ 0.20)$, $(1.4,\ 0.25)$, $(1.6,\ 0.25)$, and $(1.8,\ 0.25)$, while in the other cases, the ISCCOs are the final orbits. Also we present the PPN mass (gravitational mass) at infinity which is written as

$$M_{\mathrm{PPN,inf}} = M_*\left(1-\frac{3}{5}C_{\mathrm{s}}\right) \qquad (R\to\infty). \tag{4.13}$$

V. SUMMARY AND DISCUSSION

In this paper, we have analytically calculated the energy and the angular momentum loss rates. The conclusions are as follows.

(i) The leading finite size term of the 1PN order in dE/dt reduces $(dE/dt)_{\mathrm{N-finite}}$ by $\sim 40\%$ which is $2C_{\mathrm{s}}$ times as large as that of the Newtonian order.

(ii) The reason of the weak convergence of the finite size terms is that both of them are in the same order when we see Eq. (3.25) in the viewpoint of the series of Ω. However, the 1PN term is of $O(C_{\mathrm{s}})$ higher than the Newtonian one

in the viewpoint of the PN approximation. Therefore, the 1PN term is only by a factor of $O(C_{\rm s})$ smaller than the Newtonian one.
(iii) The leading finite size term of the 1PN order has the opposite sign of that of the Newtonian order. This is because the 1PN order terms make the self-gravity of the star stronger, then it is difficult to deform the star. Therefore, the finite size effects on the energy loss rate decrease.

Next, we discuss the definition of gravitational mass of each star. In this paper, we use the PPN mass as the gravitational mass in order to compare the equations of the energy loss rate with those of the point particle case. However, the PPN mass is appropriate only for the point particle case (i.e., as long as we ignore the tidal forces on each star of binary) [21]. In the context of present paper, we should use a gravitational mass which conserves and appropriately renormalizes the effect of the finite size of stars even if the tidal force exists. Unfortunately, no one has proposed such a mass as far as we know. To define such gravitational mass will be an unresolved problem.

Finally, we point out a problem with regard to the back reaction by gravitational radiation as follows. In this paper, we have calculated the total gravitational wave luminosity. However, it is not trivial what fraction of it contributes to the back reaction to Ω because the structure of each star (i.e., a_i in the incompressible case) is also affected due to the radiation reaction. This situation is in contrast to that for the point particle case where we only need to consider the radiation reaction to Ω. To know the change rate of Ω and a_i in the 1PN order, we need to solve the hydrodynamic equation in the 1PN order including the 3.5PN radiation reaction terms. Here, we note that from the observational point of view, we need not the total luminosity, but the back reaction of Ω. Hence, to resolve this problem seems one of the important issues in order to study the late time evolution of BNS's.

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APPENDIX A: THE DIRECT CALCULATION OF THE ANGULAR MOMENTUM LOSS RATE

In the case of the identical star binary, M_{ijk} and S_{ij} are zero. Therefore, the angular momentum loss is written as

$$\frac{dJ^i}{dt} = -\frac{2}{5c^5}\epsilon_{ijk}\left[(M_{jl}^{(2)})_{\rm tot}(M_{kl}^{(3)})_{\rm tot}\right]. \tag{A1}$$

For the quadrupole moment, the contributions from star 2 are the same as those from star 1. Then, the total quadrupole moments double those of star 1.

In the case we take in this paper, the binary rotates around the X_3 axis. Then, the angular momentum loss has only $i = 3$ component and is written as

$$\begin{aligned}\frac{dJ^3}{dt} = -\frac{2}{5c^5}\epsilon_{3jk}\Bigg[&(2D_{jl}^{(2)})(2D_{kl}^{(3)}) \\ &+\frac{1}{c^2}\Bigg\{(2D_{jl}^{(2)})\left(2V_{kl}^{(3)}+2U_{kl}^{1\to1(3)}+2U_{kl}^{2\to1(3)}+2P_{kl}^{(3)}+2Q_{kl}^{(3)}+2R_{kl}^{(3)}\right) \\ &+(2D_{kl}^{(3)})\left(2V_{jl}^{(2)}+2U_{jl}^{1\to1(2)}+2U_{jl}^{2\to1(2)}+2P_{jl}^{(2)}+2Q_{jl}^{(2)}+2R_{jl}^{(2)}\right)\Bigg\}\Bigg].\end{aligned} \tag{A2}$$

When we use the explicit forms for the quadrupole moments, the angular momentum loss is rewritten as

$$\begin{aligned}\frac{dJ^3}{dt} = -\frac{8}{5c^5}\Bigg[&2D_{11}^{(2)}D_{12}^{(3)} - 2D_{12}^{(2)}D_{11}^{(3)} \\ &+\frac{1}{c^2}\Bigg\{-D_{12}^{(2)}\Big((V_{11}-V_{22})^{(3)}+(U_{11}^{1\to1}-U_{22}^{1\to1})^{(3)}+(U_{11}^{2\to1}-U_{22}^{2\to1})^{(3)} \\ &+(P_{11}-P_{22})^{(3)}+(Q_{11}-Q_{22})^{(3)}+(R_{11}-R_{22})^{(3)}\Big)\end{aligned}$$

$$
\begin{aligned}
&+2D_{11}^{(2)}\left(V_{12}^{(3)}+U_{12}^{1\to1(3)}+U_{12}^{2\to1(3)}+P_{12}^{(3)}+Q_{12}^{(3)}+R_{12}^{(3)}\right)\\
&+D_{12}^{(3)}\Big((V_{11}-V_{22})^{(2)}+(U_{11}^{1\to1}-U_{22}^{1\to1})^{(2)}+(U_{11}^{2\to1}-U_{22}^{2\to1})^{(2)}\\
&\qquad+(P_{11}-P_{22})^{(2)}+(Q_{11}-Q_{22})^{(2)}+(R_{11}-R_{22})^{(2)}\Big)\\
&-2D_{11}^{(3)}\left(V_{12}^{(2)}+U_{12}^{1\to1(2)}+U_{12}^{2\to1(2)}+P_{12}^{(2)}+Q_{12}^{(2)}+R_{12}^{(2)}\right)\Big\}\Big],
\end{aligned}
\tag{A3}
$$

where we use the relation $D_{22}^{(i)}=-D_{11}^{(i)}$.

The contribution from the Newtonian order is

$$
2D_{11}^{(2)}D_{12}^{(3)}-2D_{12}^{(2)}D_{11}^{(3)}=16\Omega^5\hat{D}^2.
\tag{A4}
$$

The contributions from the 1PN order are written as follows:

$$
\begin{aligned}
-D_{12}^{(2)}(V_{11}-V_{22})^{(3)}+2D_{11}^{(2)}V_{12}^{(3)}&=D_{12}^{(3)}(V_{11}-V_{22})^{(2)}-2D_{11}^{(3)}V_{12}^{(2)}\\
&=32\Omega^7\hat{D}\hat{V}, &(\text{A5})\\
-D_{12}^{(2)}(U_{11}^{1\to1}-U_{22}^{1\to1})^{(3)}+2D_{11}^{(2)}U_{12}^{1\to1(3)}&=D_{12}^{(3)}(U_{11}^{1\to1}-U_{22}^{1\to1})^{(2)}-2D_{11}^{(3)}U_{12}^{1\to1(2)}\\
&=32\Omega^5\hat{D}\hat{U}^{1\to1}, &(\text{A6})\\
-D_{12}^{(2)}(U_{11}^{2\to1}-U_{22}^{2\to1})^{(3)}+2D_{11}^{(2)}U_{12}^{2\to1(3)}&=D_{12}^{(3)}(U_{11}^{2\to1}-U_{22}^{2\to1})^{(2)}-2D_{11}^{(3)}U_{12}^{2\to1(2)}\\
&=32\Omega^5\hat{D}\hat{U}^{2\to1}, &(\text{A7})\\
-D_{12}^{(2)}(P_{11}-P_{22})^{(3)}+2D_{11}^{(2)}P_{12}^{(3)}&=D_{12}^{(3)}(P_{11}-P_{22})^{(2)}-2D_{11}^{(3)}P_{12}^{(2)}\\
&=48\Omega^5\hat{D}\hat{P}, &(\text{A8})\\
-D_{12}^{(2)}(Q_{11}-Q_{22})^{(3)}+2D_{11}^{(2)}Q_{12}^{(3)}&=D_{12}^{(3)}(Q_{11}-Q_{22})^{(2)}-2D_{11}^{(3)}Q_{12}^{(2)}\\
&=-\frac{32}{7}\Omega^7\hat{D}\hat{Q}, &(\text{A9})\\
-D_{12}^{(2)}(R_{11}-R_{22})^{(3)}+2D_{11}^{(2)}R_{12}^{(3)}&=D_{12}^{(3)}(R_{11}-R_{22})^{(2)}-2D_{11}^{(3)}R_{12}^{(2)}\\
&=-\frac{256}{21}\Omega^7\hat{D}\hat{Q}. &(\text{A10})
\end{aligned}
$$

Gathering these terms, we have the angular momentum loss rate as

$$
\begin{aligned}
\frac{dJ^3}{dt}&=-\frac{8}{5c^5}\Omega_{\rm N}^5R^4M^2\Bigg[1+\frac{8}{5R^2}(a_1^2-a_2^2)+O(R^{-4})\\
&\quad+\frac{1}{c^2}\Bigg[9\pi\rho A_0-\frac{113M}{168R}+\frac{\pi\rho A_0}{R^2}\left\{\frac{296}{21}(a_1^2-a_2^2)-\frac{1}{5}(2a_1^2-a_2^2-a_3^2)\right\}\\
&\qquad+\frac{M}{840R^3}(358a_1^2+161a_2^2-2465a_3^2)+O(R^{-4})\Bigg]\Bigg], &(\text{A11})\\
&=-\frac{4M}{5c^5}\left(\frac{2M}{R}\right)^{7/2}\Bigg[1+\frac{1}{R^2}\left\{\frac{8}{5}(a_1^2-a_2^2)+\frac{3}{2}(2a_1^2-a_2^2-a_3^2)\right\}+O(R^{-4})\\
&\quad+\frac{1}{c^2}\Bigg[9\pi\rho A_0-\frac{113M}{168R}+\frac{\pi\rho A_0}{R^2}\left\{\frac{296}{21}(a_1^2-a_2^2)+\frac{133}{10}(2a_1^2-a_2^2-a_3^2)\right\}\\
&\qquad-\frac{M}{1680R^3}(2674a_1^2-2017a_2^2+3235a_3^2)+O(R^{-4})\Bigg]\Bigg]. &(\text{A12})
\end{aligned}
$$

As in the case of the energy loss rate, there remains the internal structure dependent term in the limits $a_i/R\to0$ when we rewrite the angular momentum loss rate using M_* as

$$
\begin{aligned}
\frac{dJ^3}{dt} &= -\frac{4M_*}{5c^5}\left(\frac{2M_*}{R}\right)^{7/2}\left[1+\frac{1}{R^2}\left\{\frac{8}{5}(a_1^2-a_2^2)+\frac{3}{2}(2a_1^2-a_2^2-a_3^2)\right\}+O(R^{-4})\right.\\
&\quad+\frac{1}{c^2}\left[-\frac{9\pi\rho A_0}{5}-\frac{1285M_*}{84R}-\frac{\pi\rho A_0}{5R^2}\left\{\frac{1672}{105}(a_1^2-a_2^2)+\frac{29}{2}(2a_1^2-a_2^2-a_3^2)\right\}\right.\\
&\qquad\left.\left.-\frac{M_*}{840R^3}(64274a_1^2-41171a_2^2-19645a_3^2)+O(R^{-4})\right]\right], \qquad \text{(A13)}\\
&= -\frac{4M_*}{5c^5}\left(\frac{2M_*}{R_*}\right)^{7/2}\left[1+\frac{1}{R_*^2}\left\{\frac{8}{5}(a_{1*}^2-a_{2*}^2)+\frac{3}{2}(2a_{1*}^2-a_{2*}^2-a_{3*}^2)\right\}+O(R_*^{-4})\right.\\
&\quad+\frac{1}{c^2}\left[-\frac{9\pi\rho A_{0*}}{5}-\frac{1285M_*}{84R_*}-\frac{\pi\rho A_{0*}}{5R_*^2}\left\{\frac{3016}{105}(a_{1*}^2-a_{2*}^2)+\frac{53}{2}(2a_{1*}^2-a_{2*}^2-a_{3*}^2)\right\}\right.\\
&\qquad\left.\left.-\frac{M_*}{840R_*^3}(74998a_{1*}^2-46813a_{2*}^2-22375a_{3*}^2)+O(R_*^{-4})\right]\right]. \qquad \text{(A14)}
\end{aligned}
$$

However, when we substitute the PPN mass into Eq. (A12), we have

$$
\begin{aligned}
\frac{dJ^3}{dt} &= -\frac{4M_{\rm PPN}}{5c^5}\left(\frac{2M_{\rm PPN}}{R}\right)^{7/2}\left[1+\frac{1}{R^2}\left\{\frac{8}{5}(a_1^2-a_2^2)+\frac{3}{2}(2a_1^2-a_2^2-a_3^2)\right\}+O(R^{-4})\right.\\
&\quad+\frac{1}{c^2}\left[-\frac{1285M_{\rm PPN}}{84R}-\frac{\pi\rho A_0}{5R^2}\left\{\frac{32}{21}(a_1^2-a_2^2)+(2a_1^2-a_2^2-a_3^2)\right\}\right.\\
&\qquad\left.\left.-\frac{M_{\rm PPN}}{168R^3}(13006a_1^2-8083a_2^2-3929a_3^2)+O(R^{-4})\right]\right], \qquad \text{(A15)}\\
&= -\frac{4M_{\rm PPN}}{5c^5}\left(\frac{2M_{\rm PPN}}{R_{\rm PPN}}\right)^{7/2}\left[1+\frac{1}{R_{\rm PPN}^2}\left\{\frac{8}{5}(a_{1\rm PPN}^2-a_{2\rm PPN}^2)+\frac{3}{2}(2a_{1\rm PPN}^2-a_{2\rm PPN}^2-a_{3\rm PPN}^2)\right\}\right.\\
&\qquad+O(R_{\rm PPN}^{-4})\\
&\quad+\frac{1}{c^2}\left[-\frac{1285M_{\rm PPN}}{84R_{\rm PPN}}-\frac{\pi\rho A_{0\rm PPN}}{5R_{\rm PPN}^2}\left\{\frac{256}{21}(a_{1\rm PPN}^2-a_{2\rm PPN}^2)+11(2a_{1\rm PPN}^2-a_{2\rm PPN}^2-a_{3\rm PPN}^2)\right\}\right.\\
&\qquad\left.\left.-\frac{M_{\rm PPN}}{840R_{\rm PPN}^3}(75754a_{1\rm PPN}^2-46057a_{2\rm PPN}^2-22375a_{3\rm PPN}^2)+O(R_{\rm PPN}^{-4})\right]\right], \qquad \text{(A16)}
\end{aligned}
$$

and this expression does not depend on the internal structure of the star.

If we substitute Eqs. (3.23) and/or (3.24) into Eqs. (A15) and/or (A13), we have the same equations as (3.30) and/or (3.31).

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TABLE CAPTIONS

Table I. The coordinate orbital separation R_*/a_*, axial ratios a_2/a_1 and a_3/a_1, and the frequency of gravitational wave Ω/π[Hz] of the initial and final orbits.

			Initial				Final			
$M_*/M_\odot$	M_*/c^2a_*	$M_{\rm PPN,inf}/M_\odot$	R_*/a_*	a_2/a_1	a_3/a_1	Ω/π[Hz]	R_*/a_*	a_2/a_1	a_3/a_1	Ω/π[Hz]
1.4	0.15	1.274	48.9478	0.999983	0.999971	10.00	2.6511	0.888010	0.837826	706.08
	0.20	1.232	62.9341	0.999993	0.999989	10.00	2.6278	0.918423	0.877899	1000.0
	0.25	1.190	75.2153	0.999997	0.999995	10.00	3.1321	0.965573	0.945395	1000.0
1.6	0.15	1.456	44.7585	0.999977	0.999962	10.00	2.6511	0.888010	0.837826	617.82
	0.20	1.408	57.5478	0.999991	0.999986	10.00	2.4554	0.898344	0.851178	959.53
	0.25	1.360	68.7777	0.999996	0.999993	10.00	2.8328	0.953794	0.927832	1000.0
1.8	0.15	1.638	41.3604	0.999971	0.999952	10.00	2.6511	0.888010	0.837826	549.17
	0.20	1.584	53.1789	0.999989	0.999982	10.00	2.4554	0.898344	0.851178	852.92
	0.25	1.530	63.5560	0.999995	0.999992	10.00	2.5904	0.939745	0.907524	1000.0

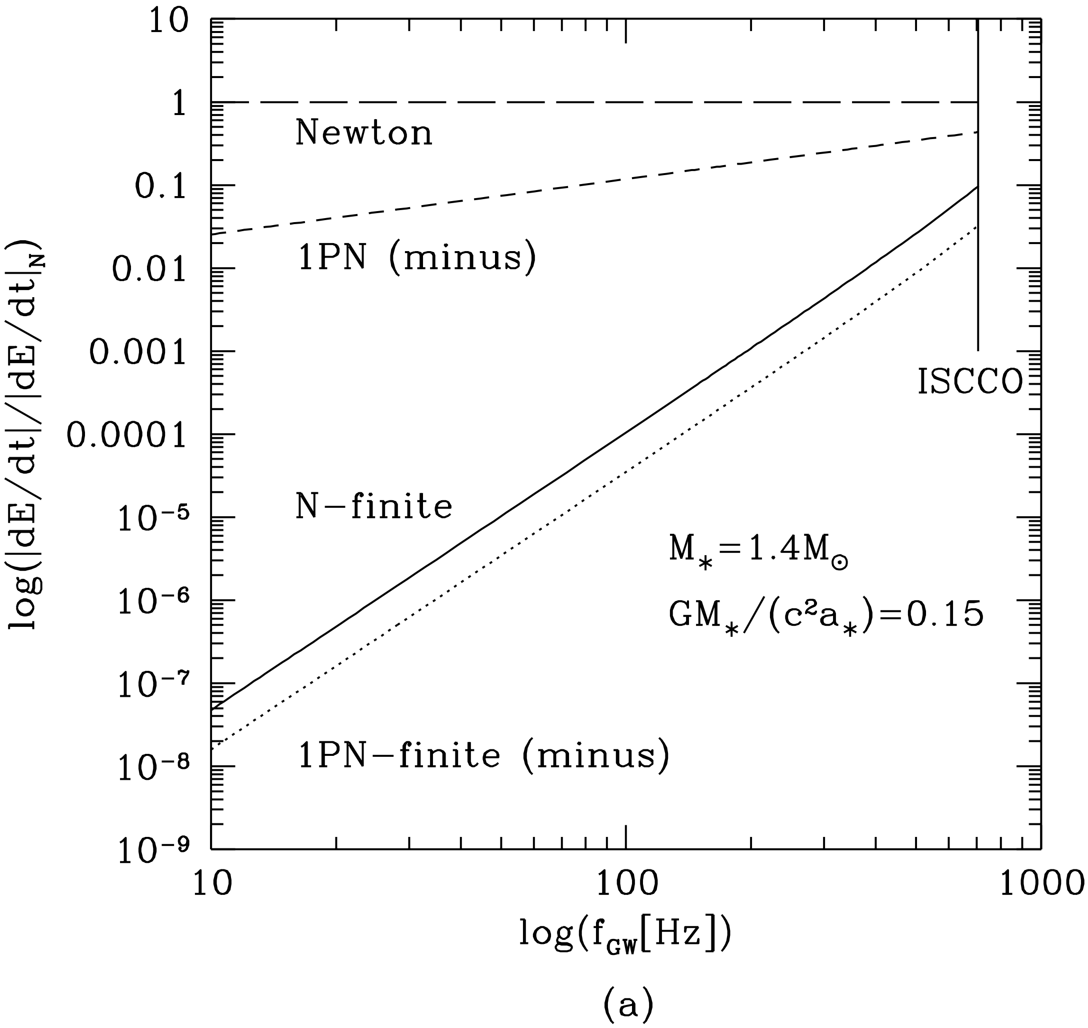
TABLE I.

FIGURE CAPTIONS

Figs.1. The energy loss rates relative to that of the Newtonian order are shown as a function of the frequency of gravitational waves $f_{\rm GW} = \Omega/\pi$. The sequences are calculated from the initial orbit (10Hz) to the final one (1000Hz or ISCCO). In these figures, we fix the conserved proper mass as $M_* = 1.4 M_\odot$ and change the compactness parameter as $C_{\rm s} = 0.15$ (a), 0.2 (b), 0.25 (c). Solid, dotted and dashed lines denote contributions from the finite size effect of the Newtonian order, that of the 1PN order and the 1PN order terms, respectively. The long-dashed line denotes the contribution from the Newtonian order term which is unity in these figures. In fig.1(a), we present the location of the energy minimum point (ISCCO) using the solid vertical line.

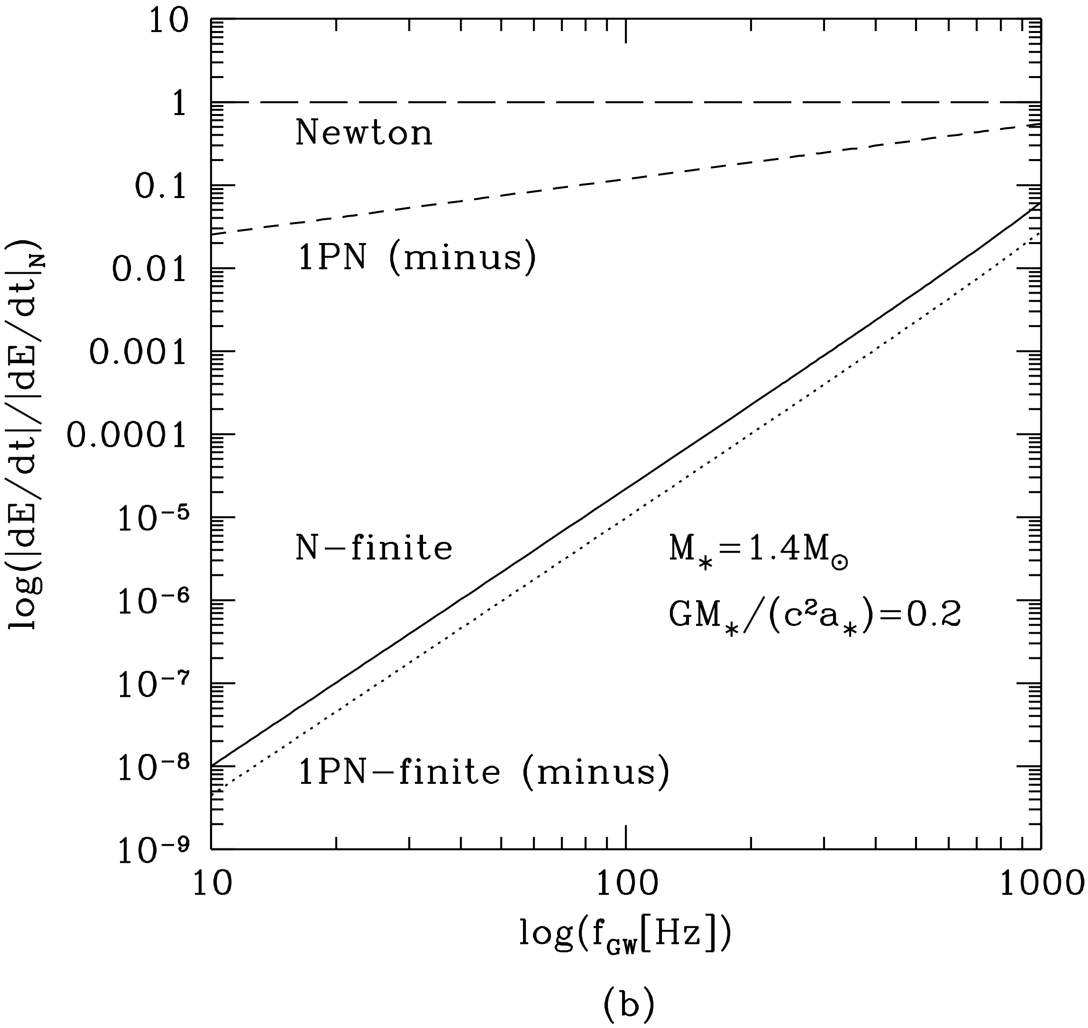
Figs.2. The conventions are the same as those in figs.1, but for M_*. Here, we fix it as $M_* = 1.6 M_\odot$ and change the compactness parameter as $C_{\rm s} = 0.15$ (a), 0.2 (b), 0.25 (c). In fig.2(a) and 2(b), we present the location of the energy minimum point (ISCCO) using the solid vertical line.

Figs.3. The conventions are the same as those in figs.1, but for M_*. Here, we fix it as $M_* = 1.8 M_\odot$ and change the compactness parameter as $C_{\rm s} = 0.15$ (a), 0.2 (b), 0.25 (c). In fig.3(a) and 3(b), we present the location of the energy minimum point (ISCCO) using the solid vertical line.



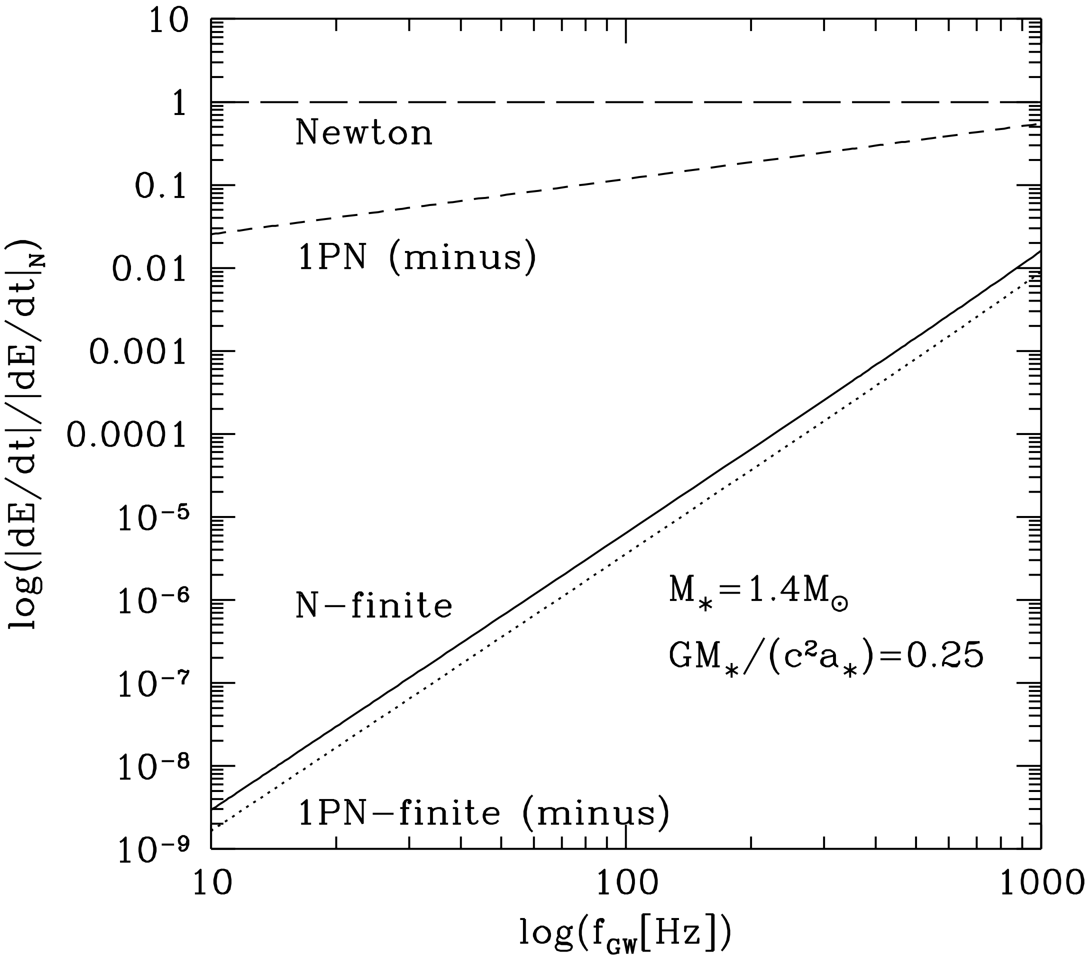
Newton
1PN (minus)
N−finite
1PN−finite (minus)
ISCCO
$M_*=1.4M_\odot$
$GM_*/(c^2a_*)=0.15$
$\log(|dE/dt|/|dE/dt|_N)$
$\log(f_{GW}[Hz])$
10
1
0.1
0.01
0.001
0.0001
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10
100
1000
(a)

Fig.1(a)



10
1
0.1
0.01
0.001
0.0001
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10
100
1000
$\log(|dE/dt|/|dE/dt|_N)$
$\log(f_{GW}[Hz])$
Newton
1PN (minus)
N−finite
1PN−finite (minus)
$M_*=1.4M_\odot$
$GM_*/(c^2a_*)=0.2$
(b)

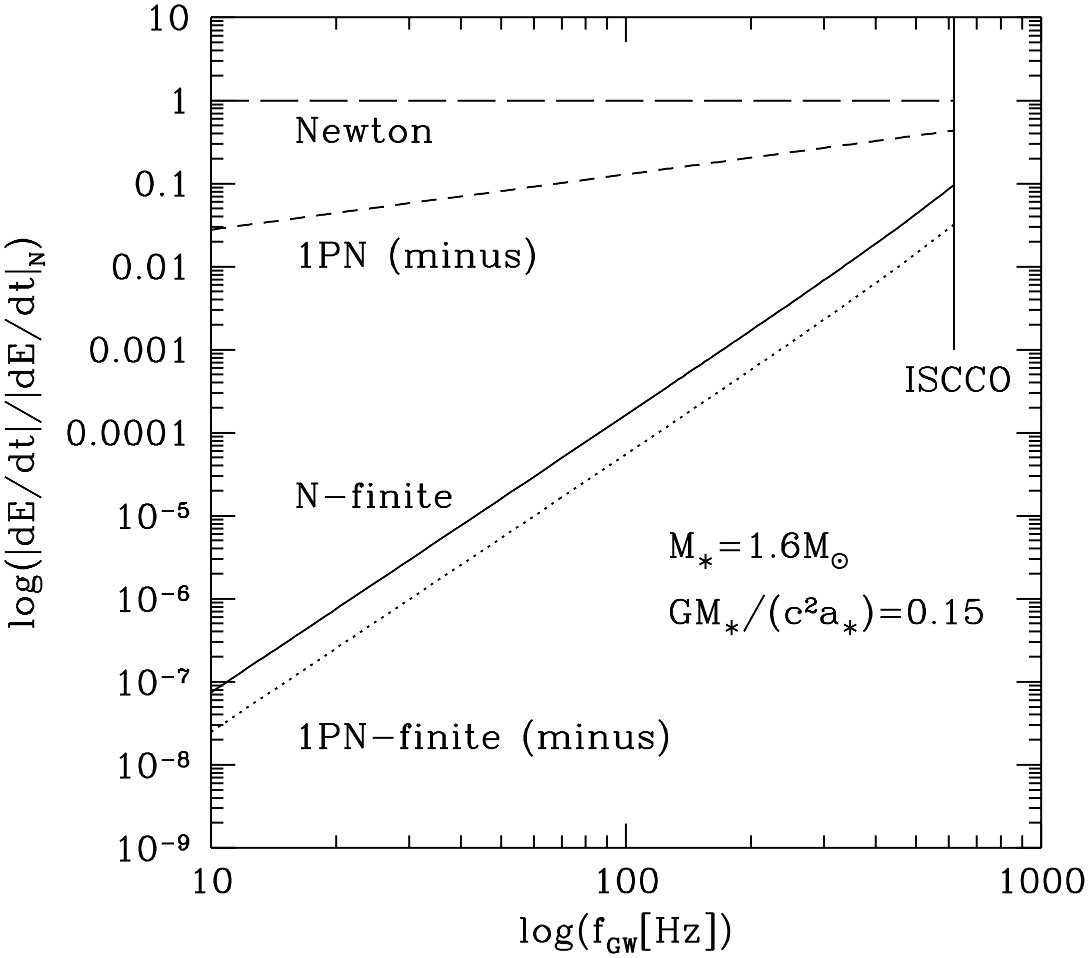
Fig.1(b)



10
1
0.1
0.01
0.001
0.0001
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10
100
1000
Newton
1PN (minus)
N−finite
1PN−finite (minus)
$M_* = 1.4 M_\odot$
$GM_*/(c^2 a_*) = 0.25$
$\log(|dE/dt|/|dE/dt|_N)$
$\log(f_{GW}[Hz])$

(c)

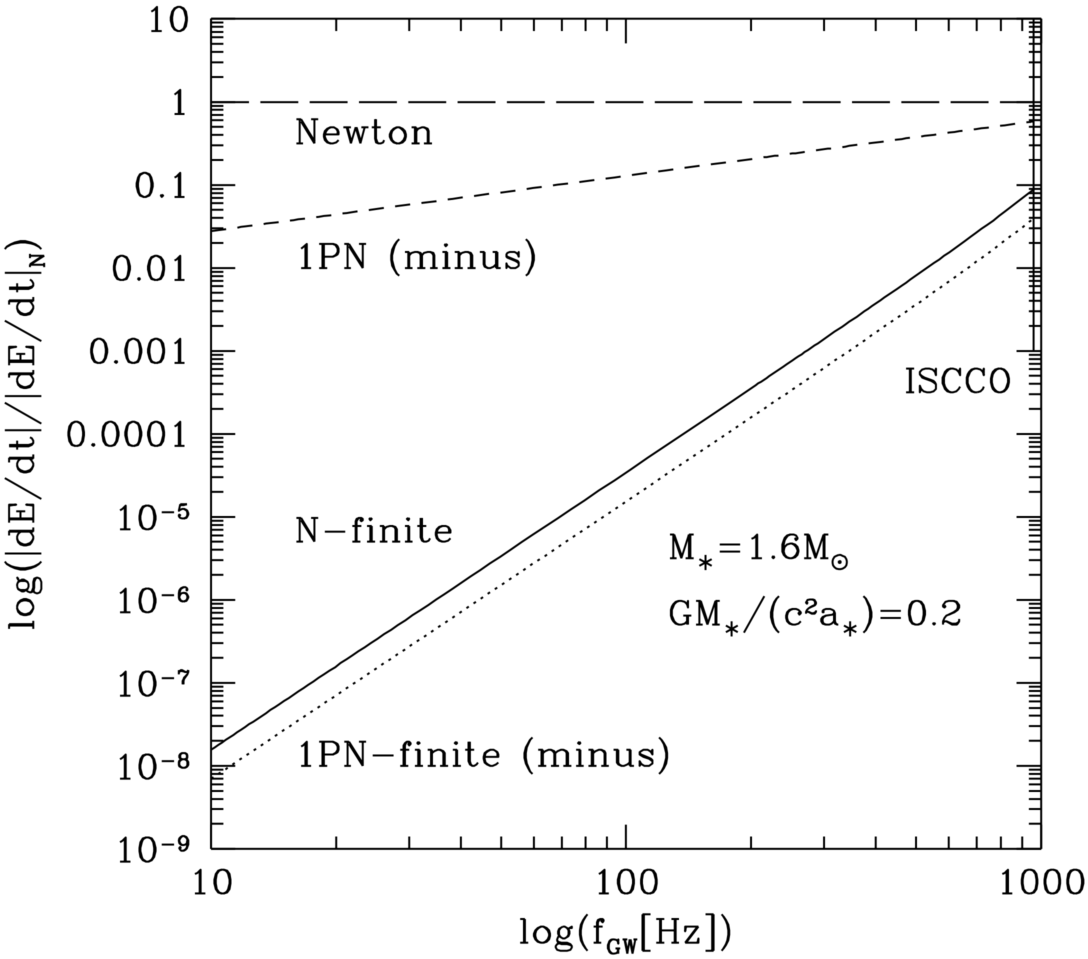
Fig.1(c)



10
1
0.1
0.01
0.001
0.0001
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10
100
1000
$\log(|dE/dt|/|dE/dt|_N)$
$\log(f_{GW}[Hz])$
Newton
1PN (minus)
ISCCO
N−finite
$M_* = 1.6M_\odot$
$GM_*/(c^2a_*) = 0.15$
1PN−finite (minus)

(a)

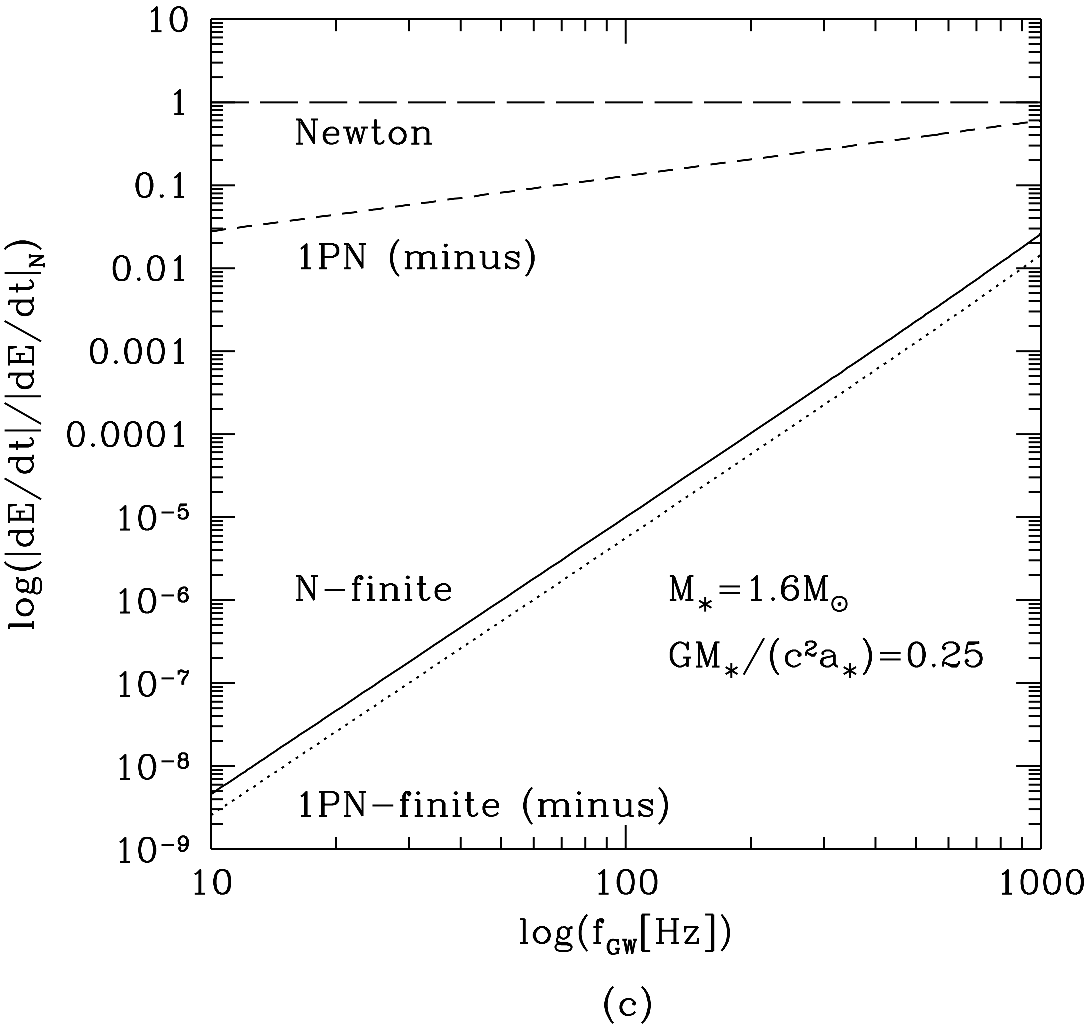
Fig.2(a)



10
1
0.1
0.01
0.001
0.0001
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10
100
1000
Newton
1PN (minus)
ISCCO
N−finite
$M_*=1.6M_\odot$
$GM_*/(c^2a_*)=0.2$
1PN−finite (minus)
$\log(|dE/dt|/|dE/dt|_N)$
$\log(f_{GW}[Hz])$

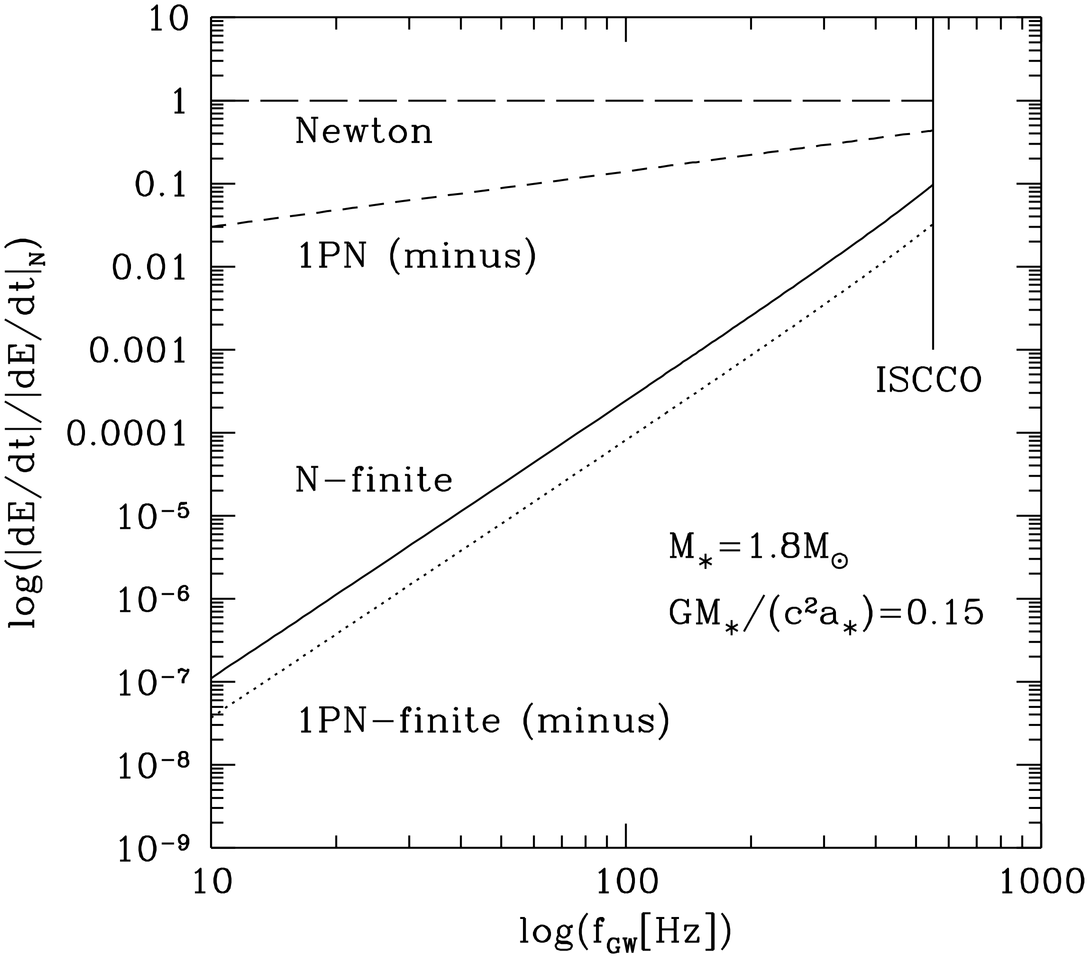
(b)

Fig.2(b)



10
1
0.1
0.01
0.001
0.0001
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10
100
1000
Newton
1PN (minus)
N−finite
1PN−finite (minus)
$M_* = 1.6 M_\odot$
$GM_*/(c^2 a_*) = 0.25$
$\log(|dE/dt|/|dE/dt|_N)$
$\log(f_{GW}[Hz])$
(c)

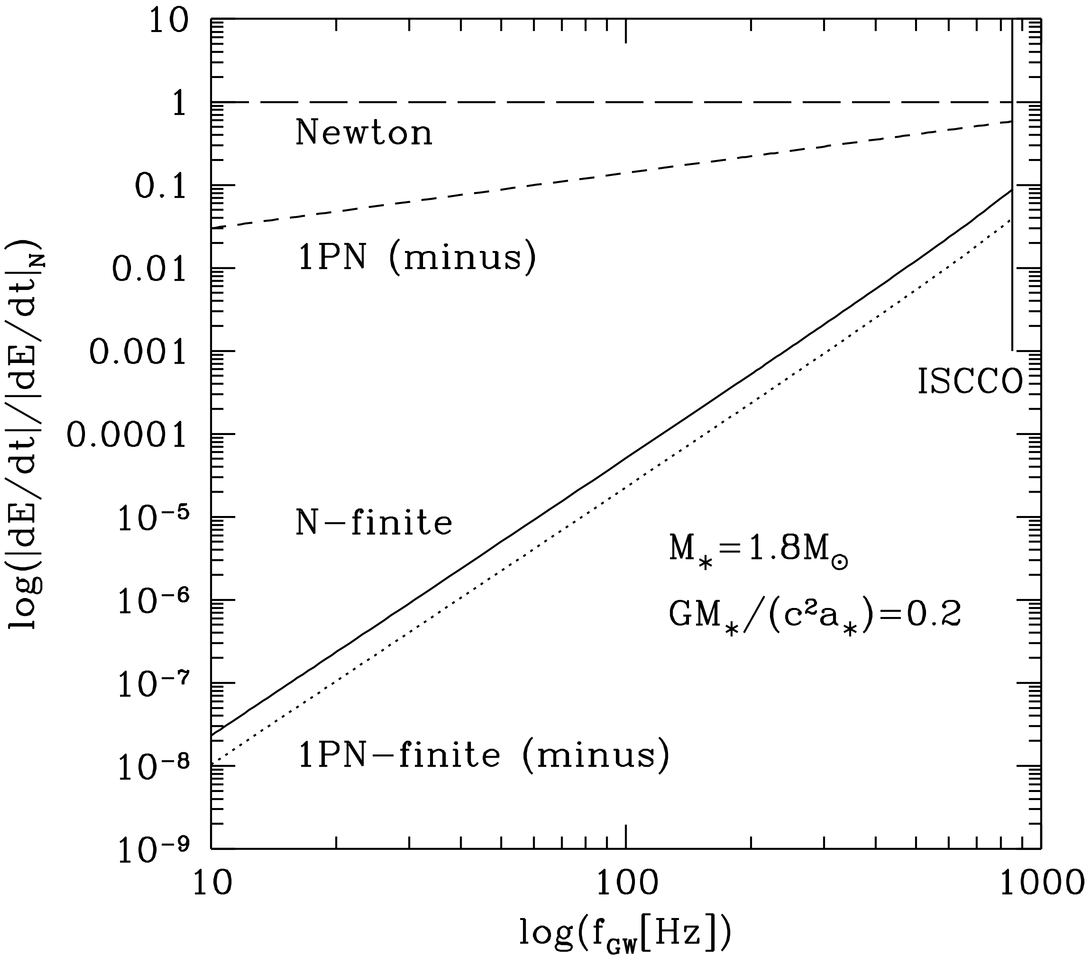
Fig.2(c)



10
1
0.1
0.01
0.001
0.0001
10⁻⁵
10⁻⁶
10⁻⁷
10⁻⁸
10⁻⁹
10
100
1000
log(|dE/dt|/|dE/dt|N)
log(fGW[Hz])
Newton
1PN (minus)
ISCCO
N−finite
M*=1.8M⊙
GM*/(c²a*)=0.15
1PN−finite (minus)

(a)

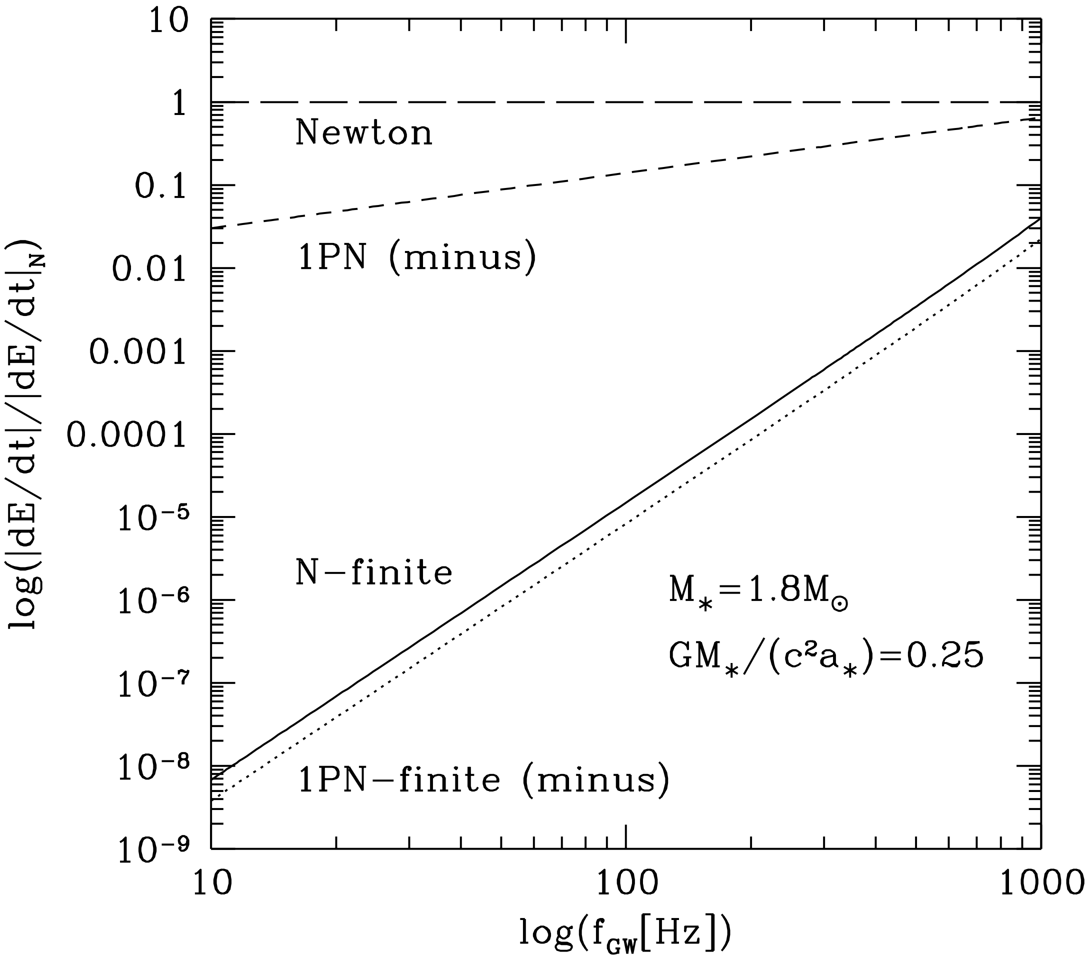
Fig.3(a)



10
1
Newton
0.1
1PN (minus)
0.01
0.001
ISCCO
0.0001
10^{-5}
N−finite
$M_* = 1.8 M_\odot$
10^{-6}
$GM_*/(c^2 a_*) = 0.2$
10^{-7}
10^{-8}
1PN−finite (minus)
10^{-9}
10
100
1000
$\log(|dE/dt|/|dE/dt|_N)$
$\log(f_{GW}[Hz])$

(b)

Fig.3(b)



10
1
0.1
0.01
0.001
0.0001
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10
100
1000
Newton
1PN (minus)
N−finite
1PN−finite (minus)
$M_*=1.8M_\odot$
$GM_*/(c^2a_*)=0.25$
$\log(|dE/dt|/|dE/dt|_N)$
$\log(f_{GW}[Hz])$

(c)

Fig.3(c)